Using Modified Adaptive Neural Fuzzy Real-time Workshop for Self-correction of the Option Pricing Model

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We propose a modified neuro-adaptive learning fuzzy system that can model, simulate, and analyze dynamic data. The system has two switches and three subsystems, which fits the need of advanced financial analysis. We can obtain the following important information: (1) the most powerful explanatory variable, (2) the length of most representative sample period, and (3) the optimal updated model. With learning mechanism, the mean absolute error will be decreased significantly.

Keywords: modified neuro-adaptive fuzzy system, self-correction of the option pricing model, real-time workshop

ACM Classification: C.1.3, C.4, D.2.9

1. INTRODUCTION

In the early 1970s, F. Black and M. Scholes made a major breakthrough in the pricing of stock options, and this has had a huge influence on the way in which market participants price and hedge options. The key assumptions underlying the Black-Scholes option pricing model (OPM) are: (1) Stock prices follow a random walk, which in turn implies that the stock price at any future time is lognormal. (2) There are no riskless arbitrage opportunities. (3) Investors can borrow or lend at the same risk-free rate of interest.

However, empirical results show that the assumed normal distribution of the logarithmic stock return cannot be supported. In addition, the stock price volatility may change randomly in practice. Finally, the estimated bias may increase or decrease as the time to maturity of the option decreases, i.e., the time-to-maturity effect (MacBeth and Merville, 1979).

To improve the model, several extensions to the traditional Black-Scholes model have been presented. Although the modified models have become more complex with additional parameters, they cannot fit the out-of- sample data better, i.e., the modified models are not robust. In addition, we cannot obtain a consistent relationship between the option price and the explanatory variables (Buraschi and Jiltso, 2006).

In this paper, we propose a modified adaptive neural fuzzy real-time workshop for self-correction of the option pricing model. Neural networks outperform the conventional Black-Scholes model when using both historical volatility and implied volatility (Blynski and Faseruk, 2006). In the online mode during the process of control, some of the new data reinforce and confirm the information contained in the previous data. Other data, however, bring new information, which could indicate a change in operating conditions, development of a fault or simply a more significant

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change in the dynamic of the process. This type of data may posses enough new information to form a new rule or to modify an existing one. The judgment of the information potential and importance of the data is based on adaptive neural fuzzy rules (Marino *et al*, 2007; Melin *et al*, 2007).

2. PROBLEMS IN ESTIMATING OPTION PRICES

Basically, there are two methods to predict option prices. One is fundamental analysis, for example, the modified Black-Scholes option pricing model. The other is technical analysis, for example, the parametric and non-parametric time series analysis.

2.1 Problems in Fundamental Analysis

Empirically, there are significant estimation errors that come from the Black-Scholes related option pricing model, the derived put-call parity, and the inverse implied volatility (to obtain the implied volatility from the Black-Scholes model). In general, the estimation errors are significant and enormous. In addition, the estimation errors are not diminished, whether for a just-in-the-money option, a just-out-of-the-money option, or a nearby option.

The only European option listed in Taiwan Stock Exchange is option for Taiwan Stock Index. In order to illustrate the possible errors, we choose four European options. They are: calls with strike prices 6300 and 6400 (hereafter 6300C and 6400C), and puts with strike prices 6300 and 6400 (hereafter 6300P and 6400P). Because they are listed in the Exchange, there is no spread problem. They were all due in June, 2004. The formula of OPM is as follows.

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

S is the stock price, X is the strike price, r is the risk-free rate of interest, T is the time to maturity in terms of the percentage of one year, σ is the annualized standard deviation of the returns of the underlying stock, and C is the option's settle price. To test against the normal distribution without specifying parameters, we use the Lilliefors test. If the test is significant at the 5% level, we reject the null hypothesis that the distribution is a normal distribution.

Table 1 shows the misspecification from Black-Scholes model for different options. "incall" is the in-the-money call, "outcall" is the out-of-the-money call, "input" is the in-the-money put, and "output" is the out-of-the-money put. The estimation error is measured as a mean absolute error (MAE), i.e., $\sum(\left|\hat{C}_t - C_t\right| / C_t)/T$. The MAEs for 6300C, 6300P, 6400C and 6400P are 32.57%, 32.20%, 48.66% and 25.19%, respectively. They are all significant and enormous.

Table 2 shows the misspecification from the derived put-call parity for different parities. "incall" and "output" have the same underlying stock, and "outcall" and "input" have the same underlying stock. According to the put-call parity, we have

$$P = C + Ke^{-rT} - S$$

The estimation error is also computed as a MAE. The MAE for K=6300/incall and output parity reaches 43.10%.

Option	Sample size	MAE (%)
K=6300/outcall	109	44.29
K=6300/incall	61	11.62
K=6300/input	109	26.61
K=6300/output	61	42.19
K=6400/outcall	39	69.90
K=6400/incall	26	16.81
K=6400/input	39	16.42
K=6400/output	26	38.35

Table 1: The Estimation Errors from the Misspecification of Black-Scholes Model

Option	Sample size	MAE (%)
6300/ outcall & input parity	109	12.37
6300/ incall & output parity	61	43.10
6400/ outcall & input parity	39	8.11
6400/ incall & output parity	26	7.00

Table 2: The Estimation Errors from the Misspecification of the Put-call Parity

Item	MAE (%)
$6300/\frac{impv(outcall)-\sigma(outcall)}{\sigma(outcall)}$	30.07
$6300/\frac{impv(incall)-\sigma(incall)}{\sigma(incall)}$	29.36
$6400/\frac{impv(outcall)-\sigma(outcall)}{\sigma(outcall)}$	61.65
$6400/\frac{impv(incall)-\sigma(incall)}{\sigma(incall)}$	37.22
$6300/\frac{impv(outcall)-impv(input)}{impv(input)}$	48.20
$6300/\frac{impv(output)-impv(incall)}{impv(incall)}$	43.58
$6400/\frac{impv(outcall)-impv(input)}{impv(input)}$	69.54
$6400/\frac{impv(output)-impv(incall)}{impv(incall)}$	36.56

Table 3: The Estimation Errors from the Inverse Problem

Concerning the inverse implied volatility, we compare three kind of pairs: (1) the difference between the volatility implied by the settle prices of option and the historical standard deviation calculated from the data of the previous 260 trading days. (2) the difference between implied volatilities calculated from call and put with the same underlying stock. We should not get different implied volatilities because the underlying stock is the same. (3) the difference between implied volatilities calculated from calls/puts with different strike prices. The only difference between the pair calls/puts is the strike price, so we should not get different implied volatilities either. For the first kind of pair, the MAE reaches 61.65%. For the second kind of pair, the MAE reaches 69.54%. The results are presented in Table 3. There are 65 trading days having both 6300C and 6400C's trading data. We use the MAE to check the third kind of pair. For example, the MAE of (impv (6400C) – impv (6300C) / impv (6300C)) is equal to 2.66%. In addition, the differences are not diminished when the days to maturity are diminished.

In addition, the derivatives of the option price model do not fit empirical data either. For example, consider Δ (delta) = $\partial C / \partial S$:

$$\frac{\partial C}{\partial S} = \frac{\partial (SN(d_1) - Xe^{-rT} N(d_2))}{\partial S}$$

$$= N(d_1) + S\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} - Xe^{-rT}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S}$$

$$= N(d_1) + Sn(d_1)\frac{\partial d_1}{\partial S} - Xe^{-rT} n(d_2)\frac{\partial d_2}{\partial S}$$

$$= N(d_1) + Sn(d_1)\frac{\partial (d_1 - d_2)}{\partial S}$$

$$= N(d_1) + Sn(d_1)\frac{\partial \sigma\sqrt{T}}{\partial S}$$

$$= N(d_1)$$

where

$$Sn(d_1) = S\frac{1}{\sqrt{2\pi}}\exp(-\frac{d_1^2}{2})$$

$$= S\frac{1}{\sqrt{2\pi}}\exp(-\frac{d_2^2 + 2\ln(S/X) + 2rT}{2})$$

$$= Sn(d_2)Xe^{-rT}/S$$

$$= Xe^{-rT}n(d_2)$$

In order to test the relationship aforementioned, we use the 6300 call option as an example. The sample period is from 21 October, 2003 to 16 June, 2004. We plot the relationship between delta $\partial C / \partial S$ and S in Figure 1. If Black-Scholes related models are correct, $\partial C / \partial S$ is equal to $N(d_1)$.

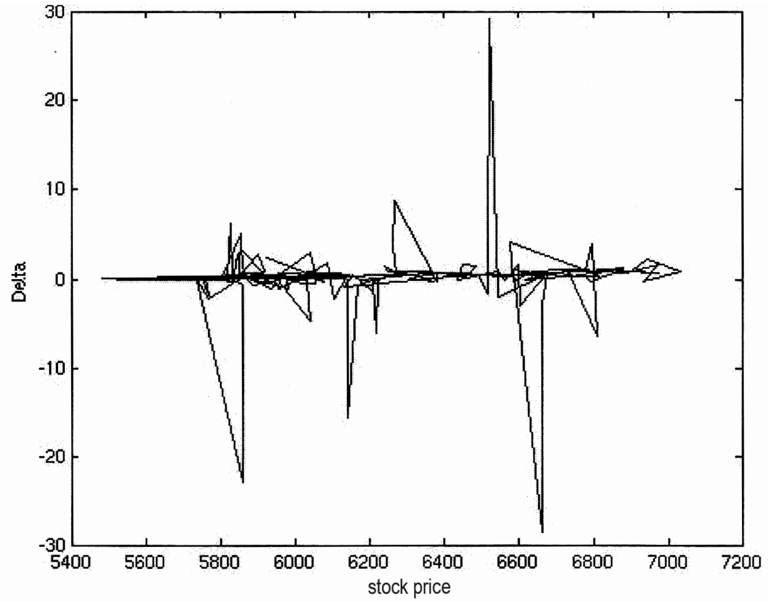

Figure 1: Variation of delta with the stock price for the 6300 call

From Figure 1, it is very obvious that $\partial C / \partial S \neq N(d_1)$, because the values of $\partial C / \partial S$ is out of the range between 0 and 1, even though it is only a small change in the underlying stock price.

2.2 Problems in Technical Analysis: De-trend vs. Prediction Power

For technical analysis, we will show that there are also some other problems using the same data. Because variables used to predict option prices are trending over time, a regression of one on the other could have a high R^2, even in the case that the two are totally unrelated. In addition, if variables in the regression are not stationary, then the standard assumptions for asymptotic analysis will not be valid. In other words, the usual t-ratios will not follow a t-distribution, so we cannot validly undertake hypothesis tests about the regression parameters. Therefore, it has been suggested that tests should be done for unit roots and cointegration. If a series contains a unit root, it is non-stationary. If the residuals have a unit root, two variables are not cointegrated. We can obtain induced stationarity by differencing once, twice, etc..

But taking differences solves only part of the statistical problems. Although we can obtain a more stationary series, both in-the-sample and out-of-the-sample errors may be increased. In addition, we will loose information by taking differences when we want to interpret the relationship between variables, i.e., we cannot use de-trend models to predict option prices. In order to show the relationship between de-trend models and estimation errors, we partition the 6300 call data into two subsets: (1) 21 October, 2003 to 17 March, 2004 (100 trading days in total); (2) 18 March, 2004 to 16 June, 2004 (65 trading days in total). The former is for the in-the-sample analysis, while the latter is for the out-of-the-sample analysis. Total square absolute errors are presented in Table 4. From Table 4, for some cases, the original form of regression is better in terms of total square absolute errors. Therefore, if a model is used mainly to predict the future behaviour, the model's prediction power is more significant than the de-trend problem.

equation	in-the-sample	out-of-the-sample
$C = \alpha_0 + \alpha_1 S$	4.2299e+05	4.9663e+05
$\Delta C = \alpha_0 + \alpha_1 \Delta S$	1.2916e+05	1.4091e+05
$\Delta^2 C = \alpha_0 + \alpha_1 \Delta^2 S$	3.0544e+05	3.0312e+05
$\Delta^3 C = \alpha_0 + \alpha_1 \Delta^3 S$	9.5433e+05	1.0102e+06
$\Delta^4 C = \alpha_0 + \alpha_1 \Delta^4 S$	3.2134e+06	3.5337e+06
$\Delta^5 C = \alpha_0 + \alpha_1 \Delta^5 S$	1.1256e+07	1.2508e+07

Table 4: Sum of Square Errors for Different De-trend Models

If we use generalized least squares (GLS) rather than ordinal least squares (OLS) to solve the problem of integration or co-integration, the best unbiased linear estimator of GLS is given by

$$\hat{\beta}_{GLS} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y$$

where $\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$; X is a matrix of observations on explanatory variables; Σ is a variance-covariance matrix of residuals, which can be obtained from OLS regression residuals; and Y is a vector of observations on the explained variable. If Σ is singular, we can then use a generalized auto-regressive conditional heteroskedasticity (GARCH) estimator instead. The GARCH (p,q) model is given by

$$\sigma_t^2 = \kappa + \alpha_1\sigma_{t-1}^2 + \ldots + \alpha_p\sigma_{t-p}^2 + \beta_1\varepsilon_{t-1}^2 + \ldots + \beta_q\varepsilon_{t-q}^2$$

where σ_t^2 is the variance of the residual, $\varepsilon_t = \sigma_t z_t$, and z_t is a standard normal distribution (Bollerslev, 1986). We can generate normally distributed random numbers from the generator of MATLAB, and we can use OLS estimated variance of the residual as proxy for σ_t^2 .

3. CRITERIA TO JUDGE THE PERFORMANCE: MINIMUM SUM OF SQUARE ERROR VS. MINIMUM SUM OF SQUARE RELATIVE ERROR

In management science, the criterion of minimum sum of square error is often used. If we use the minimum sum of square relative error criterion, we can derive the best non-linear unbiased estimator as follows.

$$Y = X\beta + u$$

where Y is a n×1 vector of observations on the explained variable, X is a n×k matrix of observations on k explanatory variables, u is a n×1 vector of residuals, and β is a k×1 vector of parameters to be estimated.

We assume that E(u) = 0, V(u) = σ^2I, $E(X'u) = 0$, $rank(X'X) = k$, and $(X'X)^{-1}$ exists. Under these assumptions, the best non-linear unbiased estimator $\hat{\beta}$ of β is given by minimizing

$$Q = \sum_{i=1}^{n}\left(\frac{y_i - \hat{\beta}_1 x_{i1} - \hat{\beta}_2 x_{i2} \ldots - \hat{\beta}_k x_{ik}}{y_i}\right)^2$$

From $\partial Q / \partial \hat{\beta}_j = 0$, we get

$$\hat{\beta}_{MSRE} = (X'AX)^{-1}X'AY$$

where A is a n×n diagonal matrix with the diagonal elements $1/y_i^2$ and $A = A' = A^{-1}$. It looks like generalized least squares $\hat{\beta}_{GLS} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y$, where $E(uu')$ is an arbitrary positive definite matrix Σ and $\Sigma^{-1/2}Y = \Sigma^{-1/2}X\beta + \Sigma^{-1/2}u$. The non-linear estimator is still the best unbiased estimator. Because the correlation between A and Y is not significant, we can ignore it.

$$E(\hat{\beta}) = \beta + E[(X'AX)^{-1}X'Au] = \beta$$

Thus $\hat{\beta}$ is unbiased.

$$V(\hat{\beta}) = E(\hat{\beta} - \beta)(\hat{\beta} - \beta)'$$
$$= \sigma^2 (X'AX)^{-1}$$

If $\tilde{\beta}$ is any other non-linear unbiased estimator of β different from $\hat{\beta}$, we will show that $V(\tilde{\beta}) > V(\hat{\beta})$. Any arbitrary non-linear unbiased estimator $\tilde{\beta}$ can be written as

$$\tilde{\beta} = \hat{\beta} + CAY$$

where CX = 0.

$$V(\tilde{\beta}) = E(\tilde{\beta} - \beta)(\tilde{\beta} - \beta)'$$
$$= \sigma^2[(X'AX)^{-1} + CC']$$

Because CC' is a positive semidefinite matrix, this proves that $\hat{\beta}$ is the best non-linear unbiased estimator.

4. ARCHITECTURE OF THE MODIFIED ADAPTIVE NEURAL FUZZY REAL-TIME WORKSHOP

In recent years grid computing technologies provide advanced computational capabilities. The dynamic data-driven application systems concept entails the ability to incorporate dynamically updated data into an executing application simulation. Such dynamic data inputs can be acquired in real-time workshop (Darema, 2005; Rubaai *et al*, 2007; Wang *et al*, 2002; Wang and Mendel, 1992). In addition, fuzzy logic controllers of the so-called Takagi-Sugeno type have gained impetus recently for applications to complex systems (Angelov, 2004; Chang *et al*, 2007; Chen and Lin, 2007; Ng *et al*, 2007; Oh *et al*, 2005; Pedrycz and Reformat, 2005; Takagi and Sugeno, 1985).

Information system now reflects a much broader subject base (Fitzgerald, 2003). For example, it has been applied to market management (Hegedus and Hopp, 2001; O'Connor and O'Keefe, 1997; Raghu *et al*, 2001) and decision making (Angelides and Paul, 1999; Avison *et al*, 2004; Saunders and Miranda, 1998; Smith-Daniels *et al*, 1996). However, there is little published research that attempts to incorporate advanced information system into advanced financial analysis. We then modify ANFIS to fit the need of advanced financial analysis.

The neuro-adaptive learning techniques have been incorporated into MATLAB's Fuzzy Logic Toolbox (ANFIS). We first create and edit the fuzzy inference system (FIS), including fuzzy rules and membership functions. We then check whether the system is the best choice, i.e., whether the training data capture the important features of all the data. By selecting the training and checking data, the modified ANFIS automatically sets the FIS parameters to be those associated with the minimum checking error (prediction error). We can also adjust the membership functions automatically. The architecture of the modified ANFIS is shown in Figure 2.

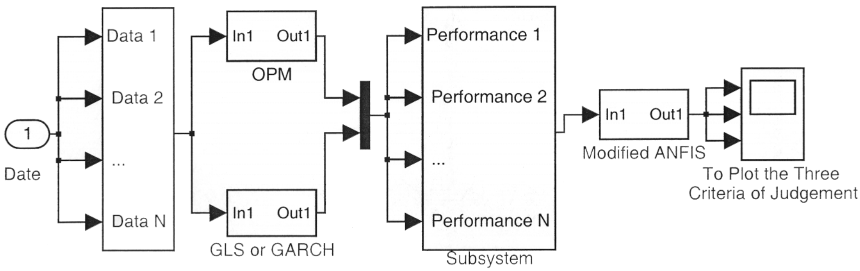

Figure 2: Architecture of Modified ANFIS

4.1 Data and Switches

Data k represents a group of online updated data. The sample size of each data group is M, where $M \geq 30$. If there are N data groups, then the total number of samples (inputs) is NM. For OPM, the exogenous variables are the theoretical variables, and the endogenous variable is the call price. For GLS (or GARCH), the exogenous variable is the underlying asset price, and the endogenous variable is the call price.

There are two switches according to different needs:

(1) Switching between the criterion of minimum sum of square error (SSE) and minimum sum of square relative error (SSRE).

(2) Switching between the incremental training and batch training. For example, there may be five inputs, i.e., 1, 2, 3, 4, and 10. If we use the batch training to get the average, we obtain that (1+2+3+4+10)/5=4. If we use the incremental training to get the average, we obtain that (1+2)/2=1.5, (1.5+3)/2=2.25, (2.25+4)/2=3.125, and (3.125+10)/2=6.5625. If the new information is more valuable, we should use the incremental training.

4.2 Subsystem of OPM

To call the following MATLAB function, we can calculate a theoretical call or put price.

[call, put] = blsprice [price, strike, rate, time, volatility, yield]

where "price" is the current price of the underlying asset; "strike" is the exercise price of the option; "rate" is the annualized risk-free rate of return over the life of the option; "time" is the time to expiration of the option expressed in years; "volatility" is the annualized standard deviation of the returns of the underlying stock; "yield" is the annualized dividend of the underlying asset over the life of the option.

4.3 Subsystem of GLS or GARCH

If the variance-covariance matrix of residuals is an arbitrary positive definite matrix Σ, instead of $\sigma^2 I$, then we transform the original model $Y=X\beta+u$ to the model

$$Y^* = X^* \beta + u^*$$

where $Y^* = \sum^{-1/2} Y;\ X^* = \sum^{-1/2} X;\ u^* = \sum^{-1/2} u$. Then $E(u^* u^{*'}) = \sum^{-1/2} E(uu') \sum^{-1/2} = I$.

The best linear unbiased estimator of b is given by

$$\hat{\beta}_{GLS} = (X^{*'} X^*)^{-1} X^{*'} Y^* = (X' \sum^{-1} X)^{-1} X' \sum^{-1} Y$$

If Σ is singular, we use the GARCH process in MATLAB instead.

[kappa, alpha, beta] = ugarch (u, p, q)

where $\sigma_t^2 = \kappa + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}^2$

$$\sum \alpha_i + \sum \beta_j < 1,\ \kappa > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0$$

$$u_t = \sigma_t v_t,\ v_t = N(0,1)$$

4.4 Subsystem of Performance

There are three criteria to judge the performance of OPM and GLS (or, GARCH): (1) the sum of square errors or the sum of square relative errors, (2) the frequency of winning, and (3) the slope of the error function. The first criterion is the most commonly used one, and a smaller sum of square errors indicates better performance. In order to obtain the frequency of winning, we count once when the square error is smaller for each sample. We then estimate the error function against time. If the slope is negative, then the estimation error is decreased as time passes by, and its performance is better than the case where the slope is positive. The results of the three criteria will be plotted in the Scope of Simulink.

5. TRAINING AND CHECKING DATA USING THE MODIFIED ANFIS

We simulate the system using the aforementioned data.

5.1 Fuzzy Rules

For OPM, we use actual call prices and theoretical call prices to calculate estimated errors. For GLS or GARCH, we use the propagation network to obtain estimated call prices, and then to calculate estimated errors. For each data set i, we can obtain an average SSE_{ij}, where j>i, and $1 \leq i, j \leq N - 1$. For the panel data, i.e., the time series data for both OPM and GLS, we sort all the average SSE_{ij}s from minimum value to maximum value.

The uniform coder for $\overline{SSE}_{ij}$ is as follows.

- IF (a value is less than or equal to 10% of all the values) THEN ($\overline{SSE}_{ij} = +5$)
- IF (a value is less than or equal to 20% of all the values) AND (a value is greater than 10% of all the values) THEN ($\overline{SSE}_{ij} = +4$)
- …
- IF (a value is less than or equal to 90% of all the values) AND (a value is greater than 80% of all the values) THEN ($\overline{SSE}_{ij} = -4$)
- IF (a value is greater than 90% of all the values) THEN ($\overline{SSE}_{ij} = -5$)

If the frequency for each sub-band is different, we can use the non-uniform coder instead. The non-uniform coder method adopts a longer step in the lowest frequency sub-band, and uses a shorter step for other ones. The non-uniform self-selective coder will improve the performance significantly. Because the relationship between $\overline{SSE}_{ij}$ and the performance is positive (defined in Section 5.2), the greater the average SSE_{ij}, the less $\overline{SSE}_{ij}$.

5.2 The Winner

We then determine which value is the winner as follows.

- Comparing the average SSE_{ij} of each data set i, if OPM's is less than GLS's, we let WIN = +1 for OPM and WIN = -1 for GLS.
- $\overline{SSE}_{ij}$ is an absolute concept and WIN is a relative concept, so they can provide different types of information.
- Using the propagation network to run the regression of SSE_{ij} s against time in each data set i, and to obtain the slope of the error function. If the slope is negative, we let SLOPE = +1, otherwise we let SLOPE = -1.
- The performance index is as follows: PERF = w_1 ($\overline{SSE}_{ij}$) + w_2 (WIN) + w_3 (SLOPE).
- Using the propagation network to run the regression $PERF_i = f(PERF_{i-1})$, we can obtain updated weights.

5.3 Results

From the system, we can obtain the following important information.

- What is the most powerful explanatory variable? For example, if the updated weights we obtain using the propagation network are $w_1 = 0$, $w_2 = .3$, and $w_3 = .7$, then the most powerful explanatory variable in the performance index is SLOPE.
- What is the optimal sample size? Because the underlying products of each listed company have different product cycles or seasonal effects, it is important to find out how long is the most representative sample period. If step i has the minimum $\overline{SSE}_{ij}$, then the most representative sample period is iM.
- What is the optimal updated model? From the modified ANFIS, we can obtain the optimal updated model by learning online.

Using the 6300 call (including both 6300/outcall and 6300/incall) as an example, we can obtain the results in Table 5. If we partition the 6300 call data into two sub-periods, GLS has the smallest MAE (2.73%) in period I (from 21 October, 2003 to 17 March, 2004), and OPM has the smallest MAE (4.29%) in period II (from 18 March, 2004 to 16 June, 2004). In addition, the distribution of residuals of GLS is a normal distribution in period I and not in period II. When compared with the results in Table 1 (44.29% for 6300/outcall and 11.62% for 6300/incall, or 32.57% for all 6300 call data), the performance is improved significantly.

	Period I		Period II	
Subsystem	OPM	GLS	OPM	GLS
MAE (%)	3.97	2.73	4.29	9.78
Lilliefors value	–	0.08	–	0.13
the critical value for the Lilliefors test*	–	0.09	–	0.11
the result of the Lilliefors test	–	normal distribution	–	not a normal distribution

Table 5: Results of the modified ANFIS

6. CONCLUSION

The modified ANFIS can learn on-line information. This learning method works similarly to that of neural networks. From the modified ANFIS, we can obtain the following important information: (1) the most powerful explanatory variable, (2) the length of most representative sample period, and (3) the optimal updated model.

Simulating the system, we find that the most powerful explanatory variable, the optimal sampling size, the optimal model are changed over time. In addition, using 6300 call as a demonstration, for period I (from 21 October, 2003 to 17 March, 2004), GLS has the smallest MAE. For period II (from 18 March, 2004 to 16 June, 2004), OPM has the smallest MAE. When compared with the results without learning, the MAEs for both periods are decreased significantly.

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REFERENCES

ANGELIDES, M.C. and PAUL, R.J. (1999): A methodology for specific, total enterprise, role-playing, intelligent gaming-simulation environment development, *Decision Support Systems*, 25: 89–108.

ANGELOV, P. (2004): A fuzzy controller with evolving structure, *Information Sciences*, 161: 21–35.

AVISON, D., JONES, J., POWELL, P. and WILSON, D. (2004): Using and validating the strategic alignment model, *Journal of Strategic Information Systems*, 13: 223–246.

BLYNSKI, L. and FASERUK, A. (2006): Comparison of the effectiveness of option price forecasting: Black-Scholes vs. simple and hybrid neural networks, *Journal of Financial Management & Analysis*, 19: 46–58.

BOLLERSLEV, T. (1989): Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31: 307–327.

BURASCHI, A. and JILTSO, A. (2006): Model uncertainty and options markets with heterogeneous beliefs, *The Journal of Finance*, 61: 2841–2898.

CHANG, P.C., WANG, Y.W. and LIU, C.H. (2007): The development of a weighted evolving fuzzy neural network for PCB sales forecasting, *Expert Systems with Applications*, 32: 86–96.

CHEN, Y.M. and LIN, C.T. (2007): Dynamic parameter optimization of evolutionary computation for on-line prediction of time series with changing dynamics, *Applied Soft Computing*, 7: 1170–1176.

DAREMA, F. (2005): Dynamic data-driven applications systems: New capabilities for application simulations and measurements, *Lecture Notes in Computer Science*, 3515: 610–615.

FITZGERALD, G. (2003): Research challenges in information systems, *International Journal of Information Management*, 23: 337–344.

HEGEDUS, M.G. and HOPP, W.J. (2001): Due date setting with supply constraints in systems using MRP, *Computers & Industrial Engineering*, 39: 293–305.

MACBETH, J.D. and MERVILLE, L.J. (1979): An empirical examination of the Black-Scholes call option pricing model, *The Journal of Finance,* 34: 1173–1186.

MARINO, F., DISTANTE, A., MAZZEO, P.L. and STELLA, E. (2007): A real-time visual inspection system for railway maintenance: automatic hexagonal-headed bolts detection, *IEEE Transactions on Systems, Man, and Cybernetics, part C – Applications and Reviews*, 37: 418–428.

MELIN, P., MANCILLA, A., LOPEZ, M. and MENDOZA, O. (2007): A hybrid modular neural network architecture with fuzzy Sugeno integration for time series forecasting, *Applied Soft Computing*, 7: 1217–1226.

NG, W.W.Y., DORADO, A., YEUNG, D.S., PEDRYCZ, W. and IZQUIERDO, E. (2007): Image classification with the use of radical basis function neural networks and the minimization of the localized generalization error, *Pattern Recognition*, 40: 19–32.

O'CONNOR, G.C. and O'KEEFE, B. (1997): Viewing the Web as a marketplace: the case of small companies, *Decision Systems*, 21: 171–183.

OH, S.K., PEDRYCZ, W. and PARK, H.S. (2005): Multi-layer hybrid fuzzy polynomial neural networks: a design in the framework of computational intelligence, *Neurocomputing*, 64: 397–431.

PEDRYCZ, W. and REFORMAT, M. (2005): Genetically optimized logic models, *Fuzzy Sets and Systems*, 150: 351–371.

RAGHU, T.S., KANNAN, P.K., RAO, H.R. and WHINSTON, A.B. (2001): Dynamic profiling of consumers for customized offerings over the Internet: a model and analysis, *Decision Support Systems*, 32: 117–134.

RUBAAI, A., OFOLI, A.R. and COBBINAH, D. (2007): DSP-based real-time implementation of a hybrid H – adaptive fuzzy tracking controller for servo-motor drives, *IEEE Transactions on Industry Applications*, 43: 476–483.

SAUNDERS, C. and MIRANDA, S. (1998): Information acquisition in group decision making, *Information & Management*, 34: 55-74.
SMITH-DANIELS, D.E., PADMAN, R. and SMITH-DANIELS, V.L. (1996): Heuristic scheduling of capital constrained projects, *Journal of Operations Management*, 14: 241–254.
TAKAGI, T. and SUGENO, M. (1985): Fuzzy identification of system and its application to modeling and control, *IEEE Transactions on System, Man, and Cybernetics*, 15: 116–132.
WANG, C.H., LIU, H.L. and LIN, T.C. (2002): Direct adaptive fuzzy-neural control with state observer and supervisory controller for unknown nonlinear dynamical systems, *IEEE Transactions on Fuzzy System*, 10: 39–49.
WANG, L.X. and MENDEL, J.M. (1992): Fuzzy basis function, universal approximation, and orthogonal least square learning, *IEEE Transactions on Neural Network*, 3: 807–814.

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