Determination of the Availability of a Shared Backup Channel

Jim M. Ng, Chor Ping Low and Ruth N. Susilo

Information Communication Institute of Singapore
School of Electrical and Electronic Engineering
Nanyang Technological University
Nanyang Ave., Singapore 639798
Email: ijmng@ntu.edu.sg

For some real-time applications, it is necessary to provide a fast switch over to the backup connection whenever the primary connection fails. With the increase in reliability of network links, it is more cost effective if the backup resources are shared among a number of connections. However, the sharing of backup resources must be controlled to ensure the users of high availability of the backup connections. In this paper, we derive a set of equations for the determination of the availability of a shared backup channel.

1. INTRODUCTION

Real-time multimedia communications in wide-area networks have been getting much attention in recent years owing to the advances in networking technology and increasing demands from multimedia applications. Although much work has been done on providing guaranteed Quality of Service (QoS) to applications, not much has been done on providing a fault-tolerant support to these applications. Traditionally, a new path will be sought only when failure occurs (Zheng and Crowcroft, 1996; Gupta and Ferrari, 1995; Balakrishnan *et al*, 1995; Ash *et al*, 1991); however, such mechanisms are not feasible for multimedia real-time applications as bandwidth may not be available and a relatively long duration may be needed to search for an alternate route. To guarantee the smooth operation of real-time communications between end users, it is necessary to pre-determine the backup connections and reserve resources so that immediate switch over can take place in case of any link failure on the primary connections. Various strategies have been proposed to provide a dedicated backup path to support guaranteed fault tolerance (Ramanathan and Shin, 1992; Han and Shin, 1997(a); Han and Shin, 1997(b)); however, with the increase in reliability of the data links, using such strategies may cause much wastage of bandwidth resources. Furthermore, as some applications may not require 100% guaranteed reliability, it is more cost effective to allow multiple connections to share the backup resources provided so that the required reliability QoS specified by each connection is satisfied.

In a wired network, a link refers to the physical connection between two neighbouring nodes and a connection refers to the set of links on the path between two end-points. Hence, whenever there is a link failure, multiple connections passing through the link will be affected. Furthermore, a

Manuscript received: 27 June 2001
Communicating Editor: Sidney A. Morris

primary connection refers to the path where data flow between two end-points during the normal operation, and a backup connection refers to the path where data flows between two-points in case its corresponding primary connection fails.

To avoid a large number of connections competing for shared resources when a link fails, control mechanism is required to limit the number of connections sharing the backup resources. However, as resources are shared, it is necessary to determine the availability of the backup resources whenever a failure occurs. Our objective is hence to provide controlled shared backup services to applications according to the reliability level requested by the applications by first determining the availability of the shared backup resources. Clearly, if an application required 100% guaranteed recovery, a dedicated path is still necessary. In the next section, the availability of a shared backup path is derived based on three assumptions: the data links are very reliable, all links provide the same amount of backup resources and all connections require the same amount of resources. In Sections 3 and 4, we will derive a new set of equations for the availability of backup paths with the last two assumptions removed. In Section 5, our current work on developing two backup provision strategies based on backup resources availability is outlined with the discussions on their relative merits. Finally, we will conclude our paper in Section 6.

2. AVAILABILITY OF A BACKUP PATH

Let us consider that a certain amount of bandwidth is reserved for backup purposes on each network link, and the bandwidth will be shared by a number of backup connections. A backup connection is always link disjoined with its primary connection such that when any link on the primary path fails, its backup path will not affected. As a number of backup connections may share bandwidth on a link, when there is a link failure affecting a number of primary connections, their backup connections may compete for the bandwidth on the links of their backup paths. In this section, we will first determine the availability of a backup connection when its primary connection fails due to one or more link failures. Let A_t be the availability of the backup path for connection t, such that

$$P(A_t) = P_{(t,1)} + P_{(t,2)} + \ldots + P_{(t,n)}, \tag{1}$$

where $P_{(t,m)}$ gives the probability of the availability of the backup path for connection t when there are m simultaneous links failure on the primary path of t, and $m = 1,\ldots, n$ where n is the number of links on the primary path of connection t.

Assuming that all links have the same failure probability, θ,

$$P_{(t,m)} = P(A_t \textit{ with m failed links})$$

$$= \binom{n}{m}\theta^m(1-\theta)^{n-m} P(A_t | m \textit{ failed links})$$

$$P(A_t) = \sum_{m=1}^{n} \binom{n}{m}\theta^m(1-\theta)^{n-m} P(A_t | m \textit{ failed links}) \tag{2}$$

With the increase in reliability of network links, the failure probability θ is getting smaller. Hence, by assuming that $\theta << 1$, $P(A_t) \approx P_{(t,1)}$, i.e. the chance of multiple link failure is very small, $P(A_t)$ can be approximated by assuming that there is only a single link failure at any one time.

Let $P(\bar{t}_i)$ denote the probability that only the i^{th} link on the primary path of the connection t fails, then,

$$P(A_t) \approx P_{(t,1)} = \sum_{i=1}^{n} P(\bar{t}_i)P(A_t | \bar{t}_i) \tag{3}$$

where $P(\bar{t}_i) = \theta(1-\theta)^{n-1}, \forall i \in [1,n]$

Let us consider a simple scenario with two primary connections C_1 and C_2, each with a backup connection b_1 and b_2 respectively as shown in Figure 1. Assume that on each link, there is a certain amount of resources set aside to be shared by all backup connections passing through the link.

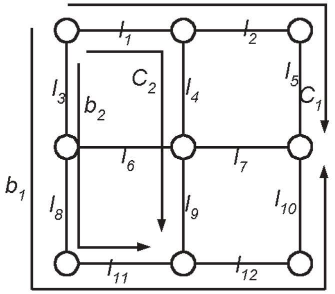

Figure 1: Homogenous backup bandwidth provisions and bandwidth requirements

For the scenario where link l_2 or l_5 fails, only connection C_1 will be affected. In such a case, the backup connection b_1 for connection C_1 can have all the backup resources available on the path b_1. However, if link l_1 fails, both of the primary connections C_1 and C_2 will be affected. As both the backup connections b_1 and b_2 pass through links l_3, l_8 and l_{11}, the two backup connections will be competing for the backup resource starting at link l_3. Assuming that both connections require the same amount of B units of bandwidth, and the amount of backup resource available on every link is B units, then the probability for connection C_i, i = 1 and 2, getting the backup resource on link l_3 is $^1/_2$. However, once a backup connection, say b_1, has failed to compete for the resource on link l_3, it will declare failure in its recovery process and it will not compete for further resources. Hence, in this case, backup connection b_2 will be able to get the resources that it needs on links l_8 and l_{11} without any further competition. The probability for both b_1 and b_2 to establish the backup connection successfully is hence $^1/_2$.

Assuming that link l_3 now has 2B units of resources for backup purpose while other links still have only B units. In this case, both backup connection b_1 and b_2 will be able to get their required resource on link l_3. However, they still need to compete for the backup resource at link l_8. As a result, the probability for both backup connections to recover successfully is again $^1/_2$.

To better illustrate our analysis, we will first make the following assumptions:

1. The network is very reliable where the failure probability θ <<1, and there will be a single link failure at any given moment.
2. All links provide the same amount of bandwidth for backup purpose.
3. All connections have the same bandwidth requirement of B units.

We will remove assumptions 2 and 3 later on in our analysis to reflect the realistic network environment.

With the above assumptions, let us define bj be the j^{th} link on the backup path and ti be the i^{th} link on the primary path of connection t respectively, and φ_{ti}^{bj} as the ratio of backup resources reserved on link l_{bj} to the amount of resources actually required when link l_{ti} fails. In other words, φ_{ti}^{bj} gives the probability of connection t being able to use the backup link l_{bj} when link l_{ti} fails. Suppose there are J connections, including connection t, with their primary paths going through link l_{ti}. Assume that among these connections, K of them share the same link, i.e. l_{bj}, on their corresponding backup paths. When link l_{ti} fails, these K connections will have to route their respective backup paths to go through link l_{bj} which has reserved WxB units of bandwidth. As a result, $\varphi_{ti}^{bj} = \frac{WB}{KB} = \frac{W}{K}$. However, as shown in the above example, when a connection fails to compete for a link on its backup path, a recovery failure message will be returned and the connection will not compete further with other connections for backup resources. Hence, the equation for φ_{ti}^{bj} needs to be refined as follows.

Let $\overline{l_i}$ denote the failure of the link l_i which is on the primary paths of some connections. We define $S(\overline{l_i}, l_j)$ as the set of connections with backup paths that will go through link l_j when their primary connections are affected by the failure of link l_i, and $S'(\overline{l_i}, l_j)$ as the set of corresponding connections that failed to get the resources on link l_j, then for a connection t, we have

$$\varphi_{ti}^{bj} = \begin{cases} \frac{\omega_{bj}}{k_{ti}^{bj}}, & if \ \frac{\omega_{bj}}{k_{ti}^{bj}} < 1 \\ 1, & if \ \frac{\omega_{bj}}{k_{ti}^{bj}} \geq 1 \end{cases} \tag{4}$$

where ω_{bj} is the multiple of B units of resources reserved on the backup link l_{bj}, and k_{ti}^{bj} is the number of connections competing for the backup resources, including connection t, when link l_{ti} fails and is defined as follows:

$$k_{ti}^{bj} = \begin{cases} \left|S(\overline{l_{ti}}, l_{bj})\right|, & for \ j = 1, \ldots, r \quad if \ \omega_{bj} \geq \left|S(\overline{l_{ti}}, l_{bj})\right| \quad (5a) \\ \left|S(\overline{l_{ti}}, l_{bj}) - S(\overline{l_{ti}}, l_{bj}) \cap \left(\bigcup_{x=1,\ldots,j-1} S'(\overline{l_{ti}}, l_{bx})\right)\right| & for \ j > r \quad otherwise \quad (5b) \end{cases}$$

Equation (5a) reflects the situation of having enough resources on the first $r-1$ links on the backup path of t to support simultaneous backup recovery for all connections which are affected by the failure of link l_{ti}, i.e. $\frac{\omega_{bj}}{k_{ti}^{bj}} \geq 1$ for $j = 1, \ldots, r\text{-}1$. At the r^{th} link on the backup connection of t, the backup resources available are not enough for the recovery of all the simultaneously connection failures caused by the failure of link l_{ti}. Equation (5b) reflects the situation where the connections that failed in competing for the backup resources in the previous links should not be considered further in the competition of resources in the current link. Note that when $\frac{\omega_{bj}}{k_{ti}^{bj}} \geq 1$, φ_{ti}^{bj} is set to 1 to reflect that the link has reserved more than enough resources for backup purpose.

As φ_{ti}^{bj} gives the probability of getting the backup resources on a link, the probability of getting the resources on a backup path is hence

$$P(A_t \mid \bar{t}_i) = \prod_{j=1}^{N} \varphi_{ti}^{bj} \tag{6}$$

where N is the number of links on the backup path of connection t. With n being the number of links on the primary connection t, we have

$$P_{(t,1)} = \sum_{i=1}^{n} P(\bar{t}_i) P(A_t \mid \bar{t}_i) = \sum_{i=1}^{n} \left[\theta (1-\theta)^{n-1} \prod_{j=1}^{N} \varphi_{ti}^{bj} \right] = \theta (1-\theta)^{n-1} \sum_{i=1}^{n} \prod_{j=1}^{N} \varphi_{ti}^{bj} \tag{7}$$

Let $P(\bar{t})$ denote the failure probability of the primary path of connection t which is caused by a link failure, and $P(A_t \mid \bar{t})$ is the probability of the availability of the backup when the connection t fails, then

$$P(A_t \mid \bar{t}) = \frac{P(A_t)}{P(\bar{t})} \approx \frac{P_{(t,1)}}{1-(1-\theta)^n} \tag{8a}$$

Substituting (7) into (8a), we get

$$P(A_t \mid \bar{t}) \approx \frac{P_{(t,1)}}{1-(1-\theta)^n} \approx \frac{\sum_{i=1}^{n}\prod_{j=1}^{N}\varphi_{ti}^{bj}}{n} \quad (8b)$$

Hence, equation (8b) approximates the availability of the backup path for a connection t.

3. HETEROGENEOUS BANDWIDTH PROVISIONS

In the above discussion, we assume that every link provides the same amount of backup resources and every connection requires the same amount of bandwidth. In this section, we will derive the availability of the backup connection where links may provide different amounts of backup resources by removing assumption 2 as stated in Section 2.

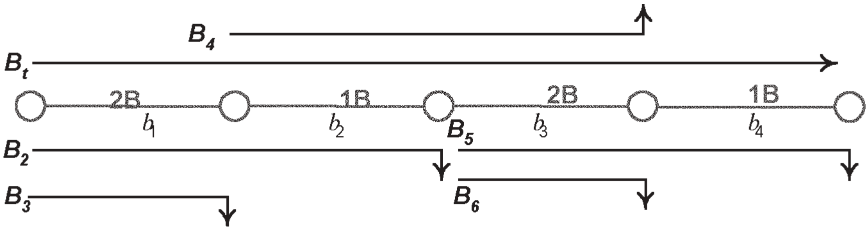

Figure 2: Heterogeneous bandwidth provisions

Let's consider the scenario given in Figure 2. Assuming that for a certain link failure in a connection, say link 1 of connection t, which is denoted as t_1, there are five other primary connections affected which backup connections, B_i, i=1,…,5, that also pass through some links of the backup connection of t, B_t, which is composed of four links (b_1 to b_4). Different amount of resources are provided on each link along this backup path as shown in Figure 2. As each backup connection requires B units of bandwidth only, there are a number of different possible combinations of connections being able to get the resource on the backup path. The sets of connections competing for the backup links l_{bi}, i=1,…,4, with the failure of link l_{t1} are given as follows:

$S(\overline{l_{t1}}, l_{b1}) = \{B_t, B_2, B_3\}$; $S(\overline{l_{t1}}, l_{b2}) = \{B_t, B_2, B_4\}$; $S(\overline{l_{t1}}, l_{b3}) = \{B_t, B_4, B_5, B_6\}$; $S(\overline{l_{t1}}, l_{b4}) = \{B_t, B_5\}$

The possible combinations of backup connections using the backup links together with their corresponding probabilities are shown in Figure 3.

To generalise the calculation of the probability for getting the backup resources in case of the failure of the i^{th} link on the primary connection t, we redefine equation (6) as

$$P(A_t \mid \overline{t_i}) = \prod_{j=1}^{N} M_j^{t_i} \quad (9)$$

where N is the number of links on the backup connection of t and $M_j^{t_i}$ is the conditional probability matrix of sets of connections getting backup link j with respect to the sets of connections getting the backup resources on backup link j-1. In $M_j^{t_i}$, the possible combinations of connections passing through link j are given in the *column* fields and the set of connections passing through the previous link in the *row* fields. Hence, the element m_{xy} in matrix $M_j^{t_i}$ specifies the probability of the set of connections given in column x being able to use the backup link j given that the set of connections specified in row y managed to get the backup resources on the previous backup link.

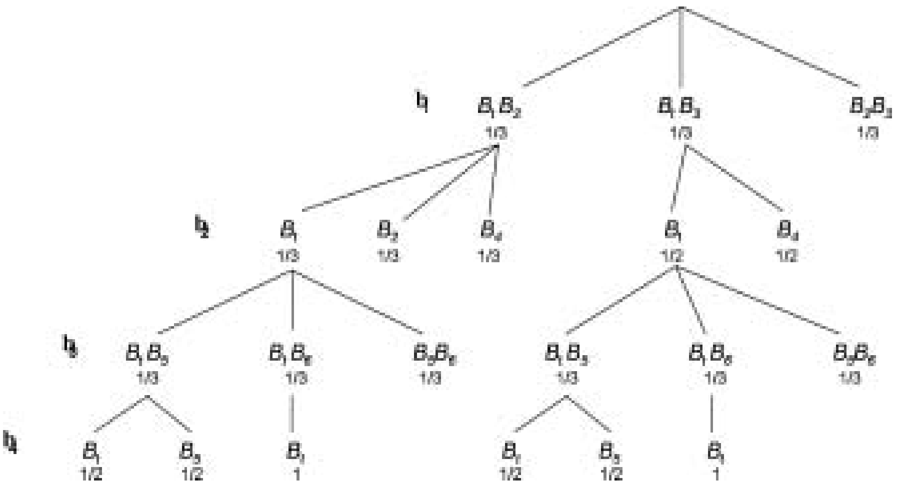

Figure 3: Probability tree for heterogeneous bandwidth provisions

As we are determining the availability of the backup resources for connection t, only the information relevant to t will be shown in the matrix for simplicity. Let us define p^l to be the l^{th} possible set of connections passing through the previous backup link j-1, and we assign values for the matrix elements of $M_j^{t_i}$ on the row corresponding to p^l, $m_j^{p^l}$, as follows:

$$m_j^{p^l} = \begin{cases} \dfrac{1}{\binom{k_{ti}^{bj,p^l}}{\omega_{bj}}} & for \quad k_{ti}^{bj,p^l} > \omega_{bj} \\ 1 & for \quad k_{ti}^{bj,p^l} \le \omega_{bj} \end{cases} \tag{10}$$

where k_{ti}^{bj,p^l} is the number of connections competing for the current backup link j given p^l, and ω_{bj} is the backup bandwidth provided by the link j. $\binom{k_{ti}^{bj,p^l}}{\omega_{bj}}$ is the number of different combinations of connections being able to use the available bandwidth ω_{bj}. Note that the connections eligible for competing for the backup link depend on the connections that have successfully gotten the backup resources on the previous links.

Referring to Figure 2, for link b_1, which provides 2B units of bandwidth, there are three connections competing $\{B_t, B_2, B_3\}$, so

$$m_1^{root} = \frac{1}{\binom{k_{t1}^{b1,root}}{\omega_{b1}}} = \frac{1}{\binom{3}{2}} = \frac{1}{3}$$, and

$$M_1^{t_1} = \begin{array}{ccc} B_tB_2 & B_tB_3 & B_2B_3 \\ \left[\frac{1}{3}\right. & \frac{1}{3} & \left.\frac{1}{3}\right] \end{array}$$

Since we are interested in the availability of the backup connection for connection t, we will consider the matrix values which contain B_t only, i.e.

$$M_1^{t_1} = \begin{array}{cc} B_tB_2 & B_tB_3 \\ \left[\frac{1}{3}\right. & \left.\frac{1}{3}\right] \end{array}$$

For link b_2 which provides 1B units of bandwidth, again there are possibly three connections, $\{B_t, B_2, B_4\}$, competing for the resources. As $M_1^{t_1}$ has two column entries, $M_2^{t_1}$ will then have two rows, each corresponds to an entry in $M_1^{t_1}$.

For the first row which corresponds to the situation where only B_tB_2 managed to get the resources in the previous link, all three connections will be competing for resources on b_2; hence,

$$m_2^{B_tB_2} = \frac{1}{\binom{k_{t1}^{b2,B_tB_2}}{\omega_{b2}}} = \frac{1}{\binom{3}{1}} = \frac{1}{3}$$

For $m_2^{B_tB_3}$ where B_2 fails to get the backup resources in the previous link, there will be only two connections competing, $\{B_t, B_4\}$; hence,

$$m_2^{B_tB_3} = \frac{1}{\binom{k_{t1}^{b2,B_tB_3}}{\omega_{b2}}} = \frac{1}{\binom{2}{1}} = \frac{1}{2}$$

As a result, we have

$$M_2^{t_1} = \begin{array}{c} \\ B_tB_2 \\ B_tB_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_t & B_2 & B_4 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \end{array}$$

As we are interested only on B_t, $M_2^{t_1}$ is simplified as

$$M_2^{t_1} = \begin{array}{c} \\ B_tB_2 \\ B_tB_3 \end{array} \begin{array}{c} B_t \\ \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix} \end{array}$$

For link b_3 which provides 2B units of bandwidth, there will be three connections, $\{B_t, B_5, B_6\}$, competing for the resources. As $M_2^{t_1}$ has only one column entry, there will be one row in $M_3^{t_1}$ which corresponds to the connection B_t which gets the backup resources successfully in the previous link. Hence,

$$m_3^{B_t} = \frac{1}{\binom{k_{t1}^{b3,B_t}}{\omega_{b3}}} = \frac{1}{\binom{3}{2}} = \frac{1}{3}$$, and

$$M_3^{t_1} = \begin{array}{c} \\ B_t \end{array}\begin{array}{cc} B_tB_5 & B_tB_6 \\ \left[\frac{1}{3}\right. & \left.\frac{1}{3}\right] \end{array}$$

Similarly, for link b_4, we have

$$M_4^{t_1} = \begin{array}{c} \\ B_tB_5 \\ B_tB_6 \end{array}\begin{array}{c} B_t \\ \left[\begin{array}{c} \frac{1}{2} \\ 1 \end{array}\right] \end{array}$$

Substituting the matrices $M_i^{t_1}$, i = 1 to 4, into equation (9), we have

$$P(A_t \mid \overline{t_1}) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{9} + \frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{6} + \frac{1}{3} \end{bmatrix} = \frac{5}{18} \cdot \frac{1}{2} = \frac{5}{36}$$

With $P(A_t \mid \bar{t}_i) = \prod_{j=1}^{N} M_j^{t_i}$, equation (3) now becomes:

$$P_{(t,1)} = \sum_{i=1}^{n} P(\bar{t}_i)P(A_t \mid \bar{t}_i) = \sum_{i=1}^{n} \left[\theta(1-\theta)^{n-1} \prod_{j=1}^{N} M_j^{t_i} \right] = \theta(1-\theta)^{n-1} \sum_{i=1}^{n} \prod_{j=1}^{N} M_j^{t_i} \quad (11)$$

Substituting (11) into (8a), equation (8b) becomes

$$P(A_t \mid \bar{t}) \approx \frac{P_{(t,1)}}{1-(1-\theta)^n} \approx \frac{\sum_{i=1}^{n} \prod_{j=1}^{N} M_j^{t_i}}{n} \quad (12)$$

Hence, equation (12) approximates the availability of the backup path for a connection t when the backup links provide different amounts of resources on every link.

4. HETEROGENEOUS BANDWIDTH REQUIREMENTS

Now we would like to relax constraint 3 where different connections may require different amounts of bandwidth. Here, equation (12) can still be applied except that the matrix entry, $m_j^{p^l}$, is to be re-defined with $m_j^{p^l} = \frac{1}{\alpha_j^{p^l}}$, where $\alpha_j^{p^l}$ gives the number of valid combinations of connections competing for the backup resources given p^l. The following example will first illustrate how $\alpha_j^{p^l}$ could be computed.

Let us assume that there are four backup connections (A,B,C,D) using a backup link bj with the failure of the i^{th} link of connection t. Backup link bj provides three units of bandwidth and the backup connections through bj require different amounts of bandwidth as shown in Figure 4.

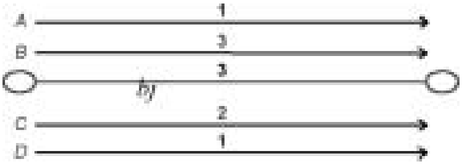

Figure 4: Heterogeneous bandwidth requirements

With $S(\overline{l}_{ti}, l_{bj}) = \{A, B, C, D\}$, we define $S_{combination}(\overline{l}_{ti}, l_{bj})$ to be the set of all possible backup connections that may pass through link bj when the i^{th} link of connection t fails given p^l, where p^l is the l^{th} possible set of connections successfully receiving the backup resources in the previous link,

and $\gamma_{ti}^{bj,p^l} = |S_{combination}(\overline{l}_{ti}, l_{bj})| = \sum_{d=1}^{k_{ti}^{bj,p^l}} \binom{k_{ti}^{bj,p^l}}{d}$, where k_{ti}^{bj,p^l}, as defined for equation (10), gives the number of backup connections competing for the backup link bj given p^l.

In Figure 4, $S_{combination}(\overline{l}_{ti}, l_{bj})$ = { {A}, {B}, {C}, {D}, {A,B}, {A,C}, {A,D}, {B,C}, {B,D}, {C,D}, {A,B,C}, {A,B,D}, {B,C,D}, {A,C,D}, {A,B,C,D}}, and the number of combinations is

$$\gamma_{ti}^{bj,p^l} = \sum_{l=1}^{4} \binom{4}{l} = 15 .$$

However, some of these combinations are not valid in practice and should be eliminated. For example, the combination {B,C} requires more than the available bandwidth and the combination {A} has adequate bandwidth to support additional backup connections; hence, these combinations are considered invalid.

Let

- B_p be the bandwidth provided by the backup link.
- B_r^k be the bandwidth required by the k^{th} set of backup connections' combination in $S_{combination}(\overline{l}_{ti}, l_{bj})$.
- $MIN_{bandwidth}^k$ be the minimum bandwidth required by a connection in $\{S(\overline{l}_{ti}, l_{bj})$ - k^{th} element of $S_{combination}(\overline{l}_{ti}, l_{bj})\}$.

A set of connections is considered invalid if:

1. The set of connections requires more bandwidth than the link can provide.
2. The spare bandwidth, (B_p - B_r^k), is equal to or larger than $MIN_{bandwidth}^k$, i.e. the spare bandwidth is enough to support additional connections.
   Such invalid combinations will therefore need to be eliminated.

The information that corresponds to our example in Figure 4 is shown in Table 1. Note that $B_p = 3$ in our example.

k	k^{th} element of $S_{combination}(\overline{l_{ti}}, l_{bj})$	B_r^k	B_p - B_r^k	S($\overline{l_{ti}}$, l_{bj})- k^{th} element of $S_{combination}(\overline{l_{ti}}, l_{bj})$	$MIN^k_{bandwidth}$
1	{A}	1	2	{B,C,D}	1
2	{B}	3	0	{A,C,D}	1
3	{C}	2	1	{A,B,D}	1
4	{D}	1	2	{A,B,C}	1
5	{A,B}	4	-1	{C,D}	1
6	{A,C}	3	0	{B,D}	1
7	{A,D}	2	1	{B,C}	2
8	{B,C}	5	-2	{A,D}	1
9	{B,D}	4	-1	{A,C}	1
10	{C,D}	3	0	{A,B}	1
11	{A,B,C}	6	-3	{D}	1
12	{A,B,D}	5	-2	{C}	2
13	{B,C,D}	6	-3	{A}	1
14	{A,C,D}	4	-1	{B}	3
15	{A,B,C,D}	7	-4	{ }	0

Table 1: Sets of possible connections for $S_{combination}(\overline{l}_{ti}, l_{bj})$

From Table 1, the sets of connections with k = 5,8,9,11,12,13,14 and 15 are invalid as B_r^k is bigger than B_p, i.e. B_p - B_r^k has a negative value indicating those combinations require more than the available bandwidth. The sets of connections with k = 1,3 and 4 are also invalid as $B_p - B_r^k$ is equal to or larger than $MIN^k_{bandwidth}$. This refers to the situation where the spare bandwidth is enough to support additional backup connections. The valid combinations of connections are shown in Table 2. $\alpha_i^{p^l}$ is then the number of connections' combinations left in Table 2, which is equal to 4 in this example.

k	k^{th} element of $S_{combination}(\overline{l_{ti}}, l_{bj})$	B_r^k	B_p - B_r^k	S($\overline{l_{ti}}$, l_{bj})- k^{th} element of $S_{combination}(\overline{l_{ti}}, l_{bj})$	$MIN^k_{bandwidth}$
1	{B}	3	0	{A,C,D}	1
2	{A,C}	3	0	{B,D}	1
3	{A,D}	2	1	{B,C}	2
4	{C,D}	3	0	{A,B}	1

Table 2: Valid sets of connections for Scombination($\overline{l}_{ti}$, l_{bj})

To summarise, $\alpha_i^{p^l}$ is formulated as follows:

Let S_k be the k^{th} entry of $S_{combination}$ ($\overline{l}_{ti}$, l_{bj}),

$$E_k = \begin{cases} -1 & \text{for } S_k \text{ with } \left(B_p - B_r^k\right) < 0 \text{ or } \left(MIN^k_{bandwidth} \neq 0 \text{ and } \left(B_p - B_r^k\right) \geq MIN^k_{bandwidth}\right) \\ 0 & \text{otherwise} \end{cases} \quad (13a)$$

$$\text{and } \gamma_{ti}^{bj,p^l} = |S_{combination}(\overline{l_{ti}}, l_{bj})| = \sum_{d=1}^{k_{ti}^{bj,p^l}} \binom{k_{ti}^{bj,p^l}}{d}, \quad (13b)$$

where k_{ti}^{bj,p^l} is the number of backup connections competing for the backup link bj given p^l which is the l^{th} set of possible connections successfully receiving the backup resources in the previous link,

then, $\alpha_j^{p^l} = \gamma_{ti}^{bj,p^l} + \sum_{k=1}^{\gamma_{ti}^{bj,p^l}} E_k$ (14)

Hence, for equation (12),

$$P(A_t \mid \bar{t}) \approx \frac{P_{(t,1)}}{1-(1-\theta)^n} \approx \frac{\sum_{i=1}^{n}\prod_{j=1}^{N} M_j^{t_i}}{n},$$

the matrix $M_j^{t_i}$ has its element redefined with $m_j^{p^l} = \frac{1}{\alpha_j^{p^l}}$.

With the new definition of $m_j^{p^l}$, we can now get the availability of the backup path for a connection t with constraints 2 and 3 relaxed.

5. CURRENT WORK ON BACKUP PROVISION STRATEGIES

With the availability of the backup path given by equation (12), we then develop two controlled shared backup strategies that can satisfy the backup availability, R_t, required by an application, such that $P(A_t \mid \bar{t}) \geq R_t$. The first strategy is to ensure that each backup link provides a minimum level of availability, ψ, by restricting the number of connections sharing a backup link. Equations (13) and (14) show that $m_j^{p^l}$ is dependent on the number of connections competing for the backup resources, k_{ti}^{bj,p^l}, and the amount of backup resources available; hence, for a new connection t, before it chooses a link for its backup, it must satisfy the condition $m_j^{p^l} \geq \psi$. From equation (12), we therefore have

$$P(A_t \mid \bar{t}) \geq \psi^N \quad (15)$$

If $m_j^{p^l} < \psi$ for some link l_{bj}, we can either re-route the backup path to use other links or increase the bandwidth on the backup link l_{bj}. However, if the availability, R_t, requested by an application on the backup connection cannot be met, i.e. $R_t > P(A_t \mid \bar{t})$, multiple backup paths have to be created. It should be noted that the actual number of connections sharing link l_{bj} is in general greater than k_{ti}^{bj,p^l} as k_{ti}^{bj,p^l} only gives the number of connections that will be competing for the backup link l_{bj} if the i^{th} link of connection t failed.

The advantage of such an approach is that the introduction of any new connections will not affect the reliability of the connections that have previously been set up, and hence avoid re-calculation of the reliability of all connections. However, by requiring $m_j^{p^l} \geq \psi$ for each link on the backup path may be too restrictive and will force the establishment of multiple backup paths resulting in a waste of resources.

To improve the backup resources utilisation, the second approach requires checking if the availability of the backup paths of the existing connections will be affected whenever a new connection is established. When a new connection t is established, the following steps have to be performed on every link on its primary path.

For each primary connection sharing the i^{th} link of t, i=1,…,n

i) check if its backup path share any links with the backup path of t;
ii) if yes, check if the availability of the backup connection can still be satisfied;
iii) if the availability cannot be satisfied, we can either increase the bandwidth reserved for

backup purpose or require the new connection t to search for an alternate backup path.

This approach can provide a better utilisation of the bandwidth but requires a large amount of computation whenever there is a new connection.

For both of the above approaches, a backup path is to be determined whenever a new connection is established to support fast recovery. The backup path can be constructed using any link disjoint path algorithm (Sidhu *et al*, 1991). For both approaches stated above, if the availability requirements cannot be satisfied, we will first attempt to increase the bandwidth reserved for backup before seeking for a new path. However, to limit the amount of resources being reserved for backup purposes, a threshold value can be set for the backup resources on each link. Hence, for the first approach, when $m_j^{p^l} \geq \psi$ on a link l_{bj} ω_{bj}, will be increased on link l_{bj} in attempt to satisfy $m_j^{p^l} \geq \psi$. If the backup resources threshold is reached, an alternate path is to be sought.

For the second approach, when the reliability requirement cannot be satisfied, one has to decide on which backup link should be chosen to increase the bandwidth. As $P(A_t \mid \bar{t})$ depends on $M_j^{t_i}$, we would choose the link with the smallest $m_j^{p^l}$ value to increase the backup resources. Again, if the backup resources on a link have reached its threshold or further increase in backup resources still cannot satisfy the availability required by a connection, an alternate path should be sought.

Besides increasing the bandwidth reserved for backup, one could also use multiple backup paths or to re-route the backup path to use links with backup resources available. Although increasing the backup resources is the most straightforward, we may not be able to increase the backup resource reservation once the threshold is reached. If multiple backup paths are used, strategies need to be derived to minimise the number of backup paths used while satisfying the reliability requirement. If re-routing is used, it is necessary to determine which links should be avoided in the construction of an alternate backup path.

Furthermore, we need to evaluate the effectiveness of the backup strategies. Basically, there are two main issues that should be considered: the complexity of the algorithms and the network resources utilisation. Clearly, our first strategy does not require any re-computation of the availability of the backup connections whenever a new connection joins; however, the condition of simply restricting the number of connections using a particular backup link may be too restrictive and does not allow a good utilisation of the network resources. The second strategy requires a more extensive re-computation but it should give a better network utilisation. Hence, it is necessary to evaluate if the gain in network utilisation is worth the extra computation effort.

It should be noted that when a link failure occurs, besides the primary connections, the backup connections passing through the link would also be affected. Hence, the sources of those affected backup connections need to be alerted so that the backup path finding mechanism will be re-activated to establish another backup path in a similar way as discussed above. Furthermore, resources that have been set aside for the previous backup paths could be released by means of a time-out mechanism.

6. CONCLUSION

With the increase in reliability of communication links, having a dedicated backup path for each connection is quite wasteful on resources. However, if backup resources are to be shared among a number of connections, it is necessary to control the number of connections sharing the resources so as to guarantee each connection some level of availability of the resources which can be specified by the applications as a QoS parameter.

In this paper, we have derived a set of equations for the determination of the availability of a shared backup path. Assuming that the failure probability of a link is small and there will be a single link failure at any given moment, the availability of a backup path is given as

$$P(A_t \mid \bar{t}) \approx \frac{\sum_{i=1}^{n} \prod_{j=1}^{N} M_j^{t_i}}{n},$$

where $M_j^{t_i}$ is the probability matrix of getting the backup resource. With this information on the availability of the backup, backup strategies are then derived to ensure the reliability required by an application can be satisfied. One strategy is based on limiting the number of connections sharing a backup link so that $m_j^{p^l} \geq \psi$ on all backup links. This will guarantee that $P(A_t \mid \bar{t}) \geq \psi^N$. If the availability required by the application is larger than ψ^N, multiple backup paths will be needed. The second strategy requires that before a backup path is to be established for a new connection, it is necessary to check that the availability requirements of all backup paths that may be affected by the failure of the new connection can still be met. Although this approach gives a better resource utilisation, a more extensive computation is required whenever a new backup connection is to be established. We are currently developing a backup establishment strategy as well as evaluating the effectiveness of both of the approaches discussed above.

REFERENCES

ZHENG W. and CROWCROFT, J. (1996): Quality-of-service routing for supporting multimedia applications, *IEEE Journal on Selected Areas in Communications*, (14)7: 1228 –1234.

GUPTA, A. and FERRARI, D. (1995): Resource partitioning for real-time communication, *IEEE/ACM Transactions on Networking*, 3(5): 501 –508.

BALAKRISHNAN, K., TIPPER, D. and MEDHI, D. (1995): Routing strategies for fault recovery in wide area packet networks, *Proc. of Military Communications Conference*, (MILCOM '95), 3: 1139 –1143.

ASH, G.R., CHANG, F. and MEDHI, D. (1991): Robust traffic design for dynamic routing networks, Proc. *INFOCOM '91*, 508 –514.

RAMANATHAN, P. and SHIN, K.G. (1992): Delivery of time-critical messages using a multiple copy approach, *ACM Trans. Computer Systems*, May, 10: 144–166.

HAN, S. and SHIN, K.G. (1997): Efficient spare-resource allocation for fast restoration of real-time channels from network component failures, *Proc. of the 18th IEEE Real-Time Systems Symposium* , 99 –108.

HAN, S. and SHIN, K.G. (1997): Fast restoration of real-time communication service from component failures in multi-hop networks, *Proc. of the ACM SIGCOMM '97*, 77–88.

SIDHU, D., NAIR, R. and ABDALLAH, S. (1991): Finding Disjoint Paths in Networks, *Proc. ACM SIGCOMM '91 Conference*, September, 43–51.

BIOGRAPHICAL NOTES

Jim Ng

Jim M. Ng received his B.Sc. (Hons) from the University of British Columbia, M.Math from the University of Waterloo, and Ph.D. from Staffordshire University in 1982, 1983 and 1999 respectively. He is presently an assistant professor in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include reliable real-time communication and mobile ad hoc networks.

Chor Ping Low

Chor Ping Low received his Ph.D. in Computer Science from the National University of Singapore (NUS) in 1994. He is currently an associate professor in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include network performance modeling and analysis, combinatorial optimization, and multicast communication. Dr. Low is a member of the IEEE Computer Society and the Association of Computing Machinery.

Chor Ping Low

Ruth N. Susilo received her DDS from University of Indonesia, her B.Sc from Bina Nusantara University, and her M.Sc from Nanyang Technological University, Singapore. She is presently working as the Quality Assurance Engineer in a private IT Solutions company in Indonesia.